

# Wilson-loop formalism for Reggeon exchange at high energy

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## Abstract

I will discuss how the non-vacuum, quark-antiquark Reggeon-exchange contribution to meson-meson elastic scattering, at high energy and low transferred momentum, can be related to the path-integral of a certain Wilson-loop expectation value over the trajectories of the exchanged fermions. Making use of this representation, I will show how a linear Regge trajectory is obtained through gauge/gravity duality and the use of minimal surfaces.

**Keywords:** Nonperturbative effects, QCD, Gauge-gravity correspondence

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## 1. Introduction

The study of soft high-energy scattering (SHES) in strong interactions ( $\sqrt{s} \gg 1$  GeV,  $\sqrt{|t|} \lesssim 1$  GeV) dates back to the mid '50s, well before the discovery of QCD. One of the key concepts in the phenomenological description of SHES is that of Regge poles, i.e., singularities in the complex-angular-momentum plane of the  $t$ -channel amplitude, corresponding in physical terms to the exchange of families of states between the colliding hadrons. The position of these singularities varies with  $t$  along the so-called Regge trajectories  $\alpha(t)$ , and governs the high-energy behaviour of the scattering amplitudes, i.e.,  $\mathcal{A}(s \rightarrow \infty, t) \sim s^{\alpha(t)}$ . The dominant trajectory in the elastic channel is called *Pomeron*, and corresponds to the exchange of states with vacuum quantum numbers, while the subleading non-vacuum trajectories are usually called *Reggeons*. The explanation of these concepts from first principles is an open problem in QCD, involving its nonperturbative (NP), strong-coupling regime.

A NP approach to SHES in the framework of QCD has been proposed some time ago [1]. This approach adopts a partonic description of hadrons over a small time-window at interaction time, over which partons do not split or annihilate and can be treated as in and out states of a scattering process. Starting from the corresponding amplitudes, one then reconstructs the hadronic amplitudes by folding with appropriate wave functions

describing the hadrons. In the case of the Pomeron exchange (PE) process, the partons travel approximately on their classical trajectories, as the energy is large, and are practically undisturbed by the diffusion process, as the momentum transfer is small, and so can be treated in an eikonal approximation [1–3]. In the case of Reggeon exchange (RE) the picture involves the exchange of a pair of valence quark and antiquark, and a different treatment is required. An approach based on the path-integral (PI) representation for the fermion propagator [4] and on analytic continuation (AC) to Euclidean space [5–7] has been suggested a few years ago [8], but a complete derivation was lacking until recently [9].

## 2. Reggeon Exchange and Wilson Loops

We briefly sketch now the derivation of a NP expression for the RE contribution to SHES amplitudes. We focus on the elastic scattering of two mesons  $M_{1,2}$ , of masses  $m_{1,2}$ , taken for simplicity with the following flavour content,  $M_1 = Q\bar{q}$ ,  $M_2 = q\bar{Q}'$ . In the soft high-energy regime, the initial momenta  $p_{1,2} = m_{1,2}u_{1,2}$ , with  $u_i$  purely longitudinal,  $u_i^2 = 1$  and  $u_1 \cdot u_2 = \cosh \chi$ , are practically unchanged by the scattering process, i.e.,  $p'_i \simeq p_i$ , with the transferred momentum  $q = p_1 - p'_1 \simeq (0, 0, \vec{q}_\perp)$ .

The starting point is adopting a description of the mesons as wave packets of transverse colourless dipoles [2], so reducing the meson-meson  $S$ -matrix,  $S_{fi}$ , to the dipole-dipole ( $dd$ )  $S$ -matrix,  $S_{fi}^{(dd)}$ ,

$$S_{fi} = \int d\mu S_{fi}^{(dd)}(\mu), \quad d\mu = d\mu_1^* d\mu_2^* d\mu_1 d\mu_2, \quad (1)$$

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where  $\mu$  denotes collectively the various degrees of freedom, and the integration measures  $d\mu_i^{(*)}$  are defined as

$$\int d\mu_i^{(*)} \equiv \int d^2 k_{i\perp} \int_0^1 d\zeta_i \sum_{s_i, t_i} \psi_i^{(*)}(\vec{k}_{i\perp}, \zeta_i), \quad (2)$$

with  $\vec{k}_{i\perp}$  and  $\zeta_i$  the transverse momentum and the longitudinal momentum fraction of the quark in meson  $i$ ,  $s_i$  and  $t_i$  the spin indices of the quark and anti-quark in meson  $i$ , respectively, and  $\psi_i$  the wave function for meson  $i$ . For later convenience we define also the wave function in coordinate space,  $\varphi_{i\,st}(\vec{R}_\perp, \zeta) = \sqrt{2\zeta(1-\zeta)2\pi} \int d^2 k_\perp e^{i\vec{k}_\perp \cdot \vec{R}_\perp} \psi_{i\,st}(\vec{k}_\perp, \zeta)$ , where  $\vec{R}_\perp$  is the transverse size of the dipole. One then performs a LSZ reduction, identifying PE with the parton-elastic process, and RE with the parton-inelastic one (Fig. 1):

$$\mathcal{S}_{fi}^{(dd)}(\mu) = \mathcal{P}^{(dd)}(\mu) + \mathcal{R}^{(dd)}(\mu). \quad (3)$$

The PE contribution has been investigated in a number of papers [1–3, 10–15], and will not be discussed here.

As regards RE, at this point one exploits the representation of the fermion propagators as PIs of Wilson lines running along the trajectories of the partons [4], and the spacetime picture of the process to identify the dominant contributions to the PI in the large  $s$ , small  $t$  regime. In the initial stage of the process a “wee” (i.e., carrying a vanishingly small fraction of longitudinal momentum) valence quark  $q$  in meson 2, and a “wee” valence antiquark  $\bar{q}$  in meson 1, enter the interaction region along the classical straight-line trajectories of the mesons, then “bend” their trajectory, and annihilate producing gluons; in the final stage of the process, these gluons produce a “wee”  $q\bar{q}$  pair, whose components rejoin the “spectator” partons to form the mesons in the final state.<sup>1</sup> As for the “spectator” partons, which carry a relevant fraction of longitudinal momentum, they travel almost undisturbed along their eikonal trajectories. This suggests that only those paths that coincide with the incoming and outgoing eikonal trajectories of the exchanged partons at early and late times contribute to the path integral.

At this point one has to work out the details, taking care of removing the internal interactions of the two mesons, that do not take part in the scattering process. Defining the RE contribution to elastic meson-meson scattering  $\mathcal{A}_R, \mathcal{R} = \int d\mu \mathcal{R}^{(dd)}(\mu) = i(2\pi)^4 \delta^{(4)}(p_f - p_i) \mathcal{A}_R$ , one obtains the following expression for  $\mathcal{A}_R$ ,

$$\mathcal{A}_R(s, t) = \lim_{\zeta_1 \rightarrow 1, \zeta_2 \rightarrow 0} \int d\tilde{\mu}_{\zeta_1, \zeta_2} \mathcal{A}_R^{(dd)}(s, t; \tilde{\mu}), \quad (4)$$

<sup>1</sup>Things can go also in the reverse order, with the production of a fermion-antifermion pair preceding the annihilation.

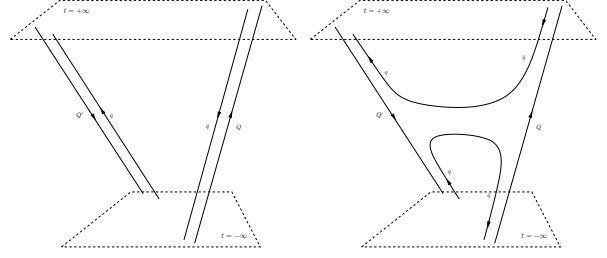


Figure 1: Space-time picture of the Pomeron-exchange (PE) process (left) and of the Reggeon-exchange (RE) process (right).

where the new integration measure  $d\tilde{\mu}_{\zeta_1, \zeta_2}$  is

$$\int d\tilde{\mu}_{\zeta_1, \zeta_2} = \int d^2 R_{1\perp} \int d^2 R_{2\perp} \int d^2 R'_{1\perp} \int d^2 R'_{2\perp} \times \sum_{t'_1, t'_2, s'_2, s_2} \rho_1 t'_1 t_1(\vec{R}_{1\perp}, \vec{R}'_{1\perp}, \zeta_1) \rho_2 s'_2 s_2(\vec{R}_{2\perp}, \vec{R}'_{2\perp}, \zeta_2), \quad (5)$$

where  $\rho_{i\,r'r}(\vec{R}_{i\perp}, \vec{R}'_{i\perp}, \zeta_i) = \sum_s \varphi_{i\,sr}^*(\vec{R}'_{i\perp}, \zeta_i) \varphi_{i\,sr}(\vec{R}_{i\perp}, \zeta_i)$  with  $\vec{R}_{i\perp}^{(\prime)}$  the dipole size in the initial (final) state, and  $\mathcal{A}_R^{(dd)}(s, t; \tilde{\mu})$  the RE contribution to  $dd$  scattering,

$$\mathcal{A}_R^{(dd)}(s, t; \tilde{\mu}) = -i2s \frac{(2\pi)^2}{m_1 m_2 N_c} \int d^2 b_\perp e^{i\vec{q}_\perp \cdot \vec{b}_\perp} \times \int \mathcal{DC}^{(\wedge)} \int \mathcal{DC}^{(\vee)} e^{i4m_q T} e^{-i(m_q - i\epsilon)(L^{(\wedge)} + L^{(\vee)})} \times S_\wedge^{t_1 s_2} [\dot{C}^{(\wedge)}; p_{\bar{q}}, p_q] S_\vee^{s'_2 t'_1} [\dot{C}^{(\vee)}; p'_q, p'_q] \mathcal{U}_C[C^{(\wedge)}, C^{(\vee)}]. \quad (6)$$

Here we have denoted  $C^{(\wedge), (\vee)} = (L^{(\wedge), (\vee)}, X^{(\wedge), (\vee)})$ ,  $\dot{C}^{(\wedge), (\vee)} = (\dot{L}^{(\wedge), (\vee)}, \dot{X}^{(\wedge), (\vee)})$ , with  $\int \mathcal{DC}^{(\wedge), (\vee)}$  defined as

$$\int \mathcal{DC}^{(\wedge), (\vee)} = \int_{2T-L_0}^{2T+L_0} dL^{(\wedge), (\vee)} \int_{x_i^{(\wedge), (\vee)}}^{x_f^{(\wedge), (\vee)}} [DX^{(\wedge), (\vee)}]; \quad (7)$$

$L^{(\wedge), (\vee)}$  is the length of path  $X^{(\wedge), (\vee)}$ , with endpoints<sup>2</sup>

$$\begin{aligned} x_i^{(\wedge)} &= -u_2 T + \frac{R_2}{2}, & x_f^{(\wedge)} &= -u_1 T + b - \frac{R_1}{2}, \\ x_i^{(\vee)} &= u_1 T + b - R'_1 + \frac{R_1}{2}, & x_f^{(\vee)} &= u_2 T + R'_2 - \frac{R_2}{2}, \end{aligned} \quad (8)$$

where we have set  $b = (0, 0, \vec{b}_\perp)$ ,  $R_{1,2}^{(\prime)} = (0, 0, \vec{R}_{1,2\perp}^{(\prime)})$ . In Eq. (6), the quantities

$$\begin{aligned} S_\wedge^{t_1 s_2} [\dot{C}^{(\wedge)}; p_{\bar{q}}, p_q] &= \frac{\bar{v}^{t_1}(p_{\bar{q}}) \mathcal{S}_{-T, -T+L} [\dot{X}^{(\wedge)}] u^{s_2}(p_q)}{2 \sqrt{\tilde{m}_q \tilde{m}_{\bar{q}}}}, \\ S_\vee^{s'_2 t'_1} [\dot{C}^{(\vee)}; p'_q, p'_q] &= \frac{\bar{u}^{s'_2}(p'_q) \mathcal{S}_{-T, -T+L'} [\dot{X}^{(\vee)}] v^{t'_1}(p'_q)}{2 \sqrt{\tilde{m}'_q \tilde{m}'_{\bar{q}}}}, \end{aligned} \quad (9)$$

<sup>2</sup>Note that they have been corrected with respect to Ref. [9].

are the normalised spin factors, where

$$\begin{aligned} S_{\eta,\nu}[\dot{X}] &= \int [\mathcal{D}\Pi] \mathcal{M}_{\eta,\nu}[\dot{X}, \Pi], \\ \mathcal{M}_{\eta,\nu}[\dot{X}, \Pi] &= \text{Texp} \left[ i \int_{\eta}^{\nu} d\tau \left( \mathbb{M}(\tau) - \Pi(\tau) \cdot \dot{X}(\tau) \right) \right], \end{aligned} \quad (10)$$

$u^s(p)$  and  $v^t(p)$  are bispinors, and  $p_q^{(\prime)} = \zeta_2^{(\prime)} p_2$ ,  $p_{\bar{q}}^{(\prime)} = (1 - \zeta_1^{(\prime)}) p_1$ . The “physical” masses of  $q$  and  $\bar{q}$  are  $\tilde{m}_q^{(\prime)} = \zeta_2^{(\prime)} m_2$ ,  $\tilde{m}_{\bar{q}}^{(\prime)} = (1 - \zeta_1^{(\prime)}) m_1$ , while  $m_q$  denotes the bare mass. Finally, the normalised Wilson loop (WL)  $\mathcal{U}_C$  is

$$\mathcal{U}_C[C^{(\wedge)}, C^{(\vee)}] \equiv \frac{\langle \mathcal{W}_C[C^{(\wedge)}, C^{(\vee)}] \rangle}{W_{T,|\vec{R}_{1\perp}|} W_{T,|\vec{R}_{2\perp}|} W_{T,|\vec{R}'_{1\perp}|} W_{T,|\vec{R}'_{2\perp}|}}, \quad (11)$$

where  $W_{T,|\vec{R}|}$  is the expectation value of a timelike  $T \times |\vec{R}|$  rectangular WL, while the WL  $\mathcal{W}_C$  runs along the path  $C$  made up of  $C^{(\wedge)}$ ,  $C^{(\vee)}$ , and the eikonal trajectories  $X_Q = u_1\tau + b + \frac{R_1}{2}$  and  $X_{\bar{Q}} = -u_2\tau - \frac{R_2}{2}$ ,  $\tau \in [-T, T]$ , of the “spectator” quarks, closed by straight-line “links” in the transverse plane to ensure gauge invariance. The length scale  $L_0$  in Eq. (7) is not specified for the time being, but it can be fixed from the experiments; the length scale  $T$  is sent to infinity at the end of the calculation.

The  $dd$  RE amplitude encodes universal properties of RE, and is basically described in terms of WLs. However, the limit of vanishing longitudinal momentum fractions of the exchanged fermions in Eq. (4), beside ensuring that only “wee” partons contribute to the process in accordance with Feynman’s picture of high energy scattering [16], leads to the endpoint behaviour of the mesonic wave functions affecting the energy dependence of the amplitude. This is due to the fact that the wave functions are typically power-like in  $\zeta$  near the endpoints, and to the fact that  $\zeta$  has to be taken to zero in the high-energy limit as  $\zeta \sim 1/\sqrt{s}$  [9].

The physical, Minkowskian amplitude in impact-parameter space can be reconstructed from a corresponding quantity in the Euclidean theory, i.e., the PI over the Euclidean trajectories of the exchanged quarks of a Euclidean WL expectation value, defined essentially as above but with the long sides forming now an angle  $\theta$  in the longitudinal plane. The Minkowskian amplitude is obtained by means of the AC  $\theta \rightarrow -i\chi$ ,  $T \rightarrow iT$ ,  $L_0 \rightarrow iL_0$  [9].

### 3. Reggeon Exchange from Gauge/Gravity Duality

The (Euclidean) WL formalism is well suited for an investigation through the gauge/gravity duality [17]. The idea behind gauge/gravity duality is that strongly coupled gauge theories can be mapped to gravity theories;

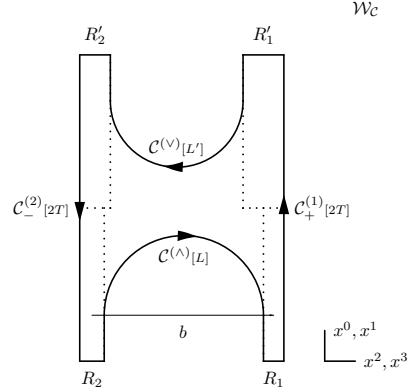


Figure 2: Schematic representation of the Wilson loop  $\mathcal{W}_C$ .

in particular, for WL expectation values the problem is reduced to finding a minimal surface (MS) in an appropriate metric having the loop contour as boundary [18]. Although a precise formulation of the duality is still lacking for confining theories, there is an important feature that the metric of the dual gravity theory is expected to have, namely the existence of a characteristic length scale (corresponding to the confinement scale) separating the near boundary region with AdS-like metric from an effectively flat region [19]. This allows to make the following approximation for the MS relevant to WL expectation values [13]. Near the boundary, the surface rises almost vertically; the horizon puts a bound on this rise, and the remaining part of the surface lives in flat space. This approximation is not sensitive to the details of the metric, so providing very general results.

Consider now heavy-light mesons of large mass, so that the typical dipole size is small, and can be neglected in a first approximation. The MS corresponding to the WL relevant to RE will be made up of four rectangular regions, living near the boundary, corresponding to the free propagation of mesons before and after the interaction, whose contribution is therefore cancelled out by the normalisation factor; and of a central strip corresponding to the exchange of a Reggeon. Up to vertical walls, that lead to the renormalisation of the quark mass, the strip lives in the flat region, and its geometry is governed by the heavy quark eikonal trajectories [8, 20]. The most important contributions will come therefore from exchanged-quark trajectories lying on the corresponding helicoid of opening angle  $\theta$ . In the saddle-point approximation, one retains only the maximal contribution, corresponding to the trajectories that minimise the Euclidean “effective action”,

$$S_{\text{eff}, E} \equiv \frac{1}{2\pi\alpha'_{\text{eff}}} A + \hat{m}_q (L^{(\vee)} + L^{(\wedge)} - 4T), \quad (12)$$

that includes contributions from both the area  $A$  of the surface and the length of its boundaries, subject to the condition of joining smoothly the incoming and outgoing eikonal trajectories. An exact solution to this variational problem exists [20], that is independent of  $T$  and  $L_0$ , but that however is in implicit form and thus not suitable for our purposes. Since we have to perform the AC, we need to know explicitly the dependence in  $\theta$ : this can be achieved making the approximation of small  $\theta$ . After going back to Minkowski space-time, and in the limit of small (renormalised) quark mass  $\hat{m}_q$ , one finds [20]

$$S_{\text{eff}, M} \simeq \frac{b^2}{4\alpha'_{\text{eff}}\chi} - \frac{4b\hat{m}_q}{\chi} + 2\pi^2\alpha'_{\text{eff}}\hat{m}_q^2, \quad (13)$$

where  $S_{\text{eff}, M} \equiv S_{\text{eff}, E}|_{\theta \rightarrow -i\chi}$ , and  $\alpha'_{\text{eff}}$  is the effective string tension corresponding to the confining background.

In order to obtain the complete expression for the impact-parameter amplitude in the saddle-point approximation, one should take into account also spin effects, and quantum fluctuations around the solution; these terms are however not completely under control at the moment, and will be neglected in a first approximation, so that the impact-parameter amplitude takes the Gaussian form  $a(\chi, \vec{b}) \approx e^{-S_{\text{eff}, M}}$ . One then obtains, after Fourier transform, the following  $dd$  scattering amplitude,  $\mathcal{A}_R^{(dd)}(s, t) \approx \mathcal{T}_0(\chi, t) + \hat{m}_q \mathcal{T}_1(\chi, t) + \mathcal{O}(\hat{m}_q^2)$ , where<sup>3</sup>

$$\begin{aligned} \mathcal{T}_0(\chi, t) &= e^{-\alpha'_{\text{eff}}\chi q^2}, \\ \mathcal{T}_1(\chi, t) &= 8\sqrt{\pi\alpha'_{\text{eff}}}\frac{\partial}{\partial\chi}\left[\sqrt{\chi}\tilde{I}_0\left(\alpha'_{\text{eff}}\chi\frac{q^2}{2}\right)\right], \end{aligned} \quad (14)$$

with  $\tilde{I}_0(z) = e^{-z}I_0(z)$  ( $I_0$ : modified Bessel function).

Inspecting the singularity structure in the complex-angular-momentum plane by means of a Mellin transform, one finds a linear Regge trajectory: the linearity and the slope are not affected by prefactors of the type  $s^{\delta\alpha}\chi^{n_\chi}b^{n_b}$ , that could come from the neglected terms, and is therefore a robust result. In particular, for massless quarks one recovers the result of Ref. [8], exactly corresponding to a Regge pole, while for small but nonzero  $\hat{m}_q$  the singularity contains also a logarithmic branch point [20]. Notice that the slope is equal to the inverse string tension  $\alpha'_{\text{eff}}$ : this provides a first bridge between the WL formalism and the usual Regge picture of the exchange between the hadrons of states lying on the same linear trajectory in the  $(J, m^2)$  plane.

<sup>3</sup>A factor  $(2\alpha'_{\text{eff}}\chi)^{-1}$  has been inserted “by hand” in order to remove an extra logarithmic prefactor: this however does not change our conclusions.

## 4. Conclusions

A formalism based on WLs is now available for the NP study of RE in SHES. The AC to Euclidean space allows to apply NP techniques to study this problem. The results obtained to leading order through gauge/gravity duality are encouraging, and it would be interesting to compute the effects of quantum fluctuations around the saddle point, and the effects of dynamical quarks.

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